An Accurate and Efficient Hybrid Method for the Calculation of the Equivalent Capacitance of an Arbitrary Shaped Coil

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In this paper we present a novel hybrid method to efficiently and accurately calculate the equivalent capacitance of an arbitrary shaped coil with circular cross section by using finite element method in 3D. To avoid the high mesh effort and the computing time, a parallel-electrode capacitor as the basic cell is used to approximate the circular wires. According to the published analytical formula, an equivalent relative permittivity for the basic cell can be obtained to ensure the accuracy of the presented method. Since the formulas are derived under different physic insights, the optimal equivalent relative permittivity for different winding methods of the coil has been determined. Finally, the efficiency and the accuracy of the presented method are validated through the numerical results.

***Index Terms*—Capacitance, Numerical simulation, Finite element analysis.**

I. Introduction

Coils are very common electrical components and widely used in almost all application fields related to the magnetic field. The parasitic capacitance, which is an essential parameter for the determination of the equivalent capacitance of a coil, can be calculated analytically [1]-[6]. One simple parallel-plate capacitor model for the approximation of the equivalent capacitance of a coil is described in [1] and another two formulas to calculate the parasitic capacitances between the adjacent turns have been addressed in [2]-[5]. A systematic summary of the known approaches is presented in [6]. These papers have shown some interesting physical insights, however, they are not suitable for an arbitrary shaped coil. Moreover, the edge effect and the capacitances between non-adjacent turns cannot be considered. Compared to the analytical method, the finite element method (FEM) can model the arbitrary shaped coil and offer very accurate results [7]-[9]. However, it is a time consuming process. Thus, it is interesting to find a compromise between the analytical method and the FEM to calculate the equivalent capacitance of a coil accurately and efficiently.

In order to improve the mesh efficiency in FEM, a parallel-electrode capacitor (PEC) model as a basic cell is used to model the turns of the coil and, thus, a great number of elements for the circular cross-section and the air gaps can be highly reduced. Moreover, an equivalent relative permittivity (ERP) of the basic cell is presented to calculate the equivalent capacitance of the coil accurately. Another advantage is that less number of degrees of freedom (DOFs) of the system matrix reduce the computer effort significantly. The accuracy and efficiency of the presented method are successfully demonstrated through the numerical results.

II. Electrostatic Formulation

According to the definition of electrical field strength and the differential form of Gauss's law, the Poisson equation can be stated by the following expression:

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0\varepsilon_r} \quad , \tag{1}$$

where ϕ is the electrical potential and ρ the charge density. ε_0 and ε_r are the permittivity of the vacuum and the relative permittivity of the medium, respectively.

Introducing the second-order scalar shape function for ϕ and applying the Galerkin method, the system matrix can be described as follows:

$$[K]\{\phi\} = \{b\} \quad , \tag{2}$$

where K is the stiffness matrix and b the source vector.

III. Analytical Formulations

To analytically calculate the equivalent capacitance of a coil, it is necessary to obtain the capacitance between adjacent turns (turn-to-turn capacitance). Normally, the orthogonal winding method or the orthocyclic winding method is used for wrapping the coil (Fig. 1).

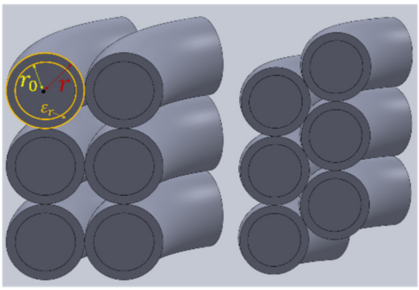

Fig. 1. Orthogonal winding method (left) and orthocyclic winding method (right).

It is assumed that the capacitance per unit length of the wire is considered, as the shape of the coil is arbitrary. In theory, the basic formulation presented by Massarini (M-formula) in [3] can be used for both winding methods except using the different integration path. The turn-to-turn capacitance per unit length for the orthogonal winding is expressed as follows:

$$C_{tt} = \frac{2\varepsilon_0\varepsilon_r \operatorname{atan}\left(\left(\sqrt{\ln(\frac{r}{r_0})} + 2\varepsilon_r\right)\Big/\sqrt{\ln(\frac{r}{r_0})}\right)}{\sqrt{\ln^2\left(\frac{r}{r_0}\right) + 2\varepsilon_r \ln(\frac{r}{r_0})}} \quad , \tag{3}$$

where ε_r is the relative permittivity of the wire's insulation. r and r_0 are the radii of wire with and without insulation, respectively. Since this digest is limited to 2 pages, the turn-to-turn capacitance C_{tt} for both winding methods described by Koch (K-Formula) in [2] will be given in the full paper clearly.

To reduce the mesh effort, a PEC model is presented (Fig. 2). The red surface of each wire has a constant electrical potential denoted with the yellow number. Moreover, the electrical potentials along the turns are assumed to be linear [6]. Then, each circular wire can be replaced by a rectangular surface with the same electrical potential. Furthermore, an ERP ε_{equi} between two flat surfaces is calculated to ensure the capacitance per unit length of this model equal to the value of the original one's. Therefore, a basic capacitance model with the simplified geometry has been obtained. By using the FEM simulation the equivalent capacitance of an arbitrary shaped coil can be calculated efficiently and accurately.

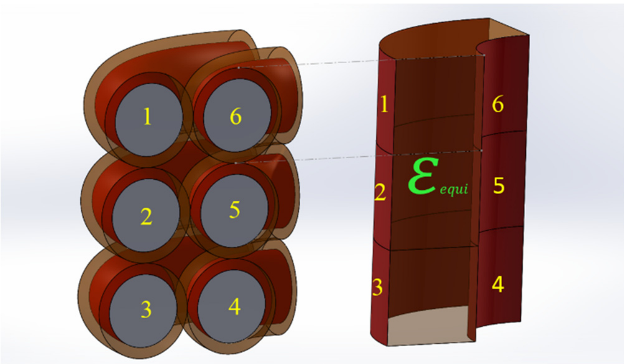

Fig. 2. The Orthogonal turns (left) and the PEC model (right).

IV. Numerical Results

A. Cylindrical coil

To verify the presented method, a cylindrical coil is firstly arranged in two dimensional axisymmetric model. The coil has two layers and the number of turns of each layer is varied from 1 to 96. The results are shown in Fig. 3.

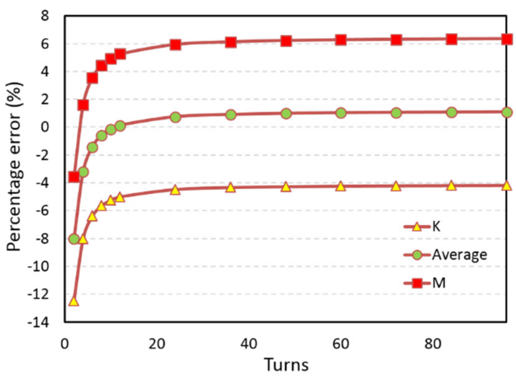

Fig. 3. Percentage error for the orthogonal winding by using the presented formulas.

The results have shown that an optimal ERP for the orthogonal winding method is the average value calculated by the K and M formulas.

B. Arbitrary shaped coil

The efficiency and the accuracy of the presented approach is demonstrated on the example shown in Fig. 4.

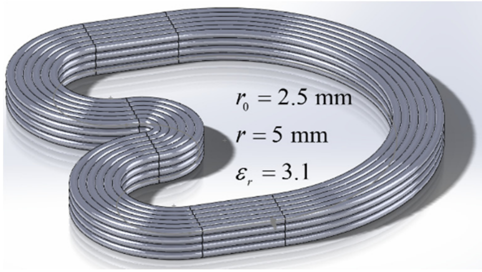

Fig. 4. An arbitrary shaped coil with 6 layers and 4 turns of each layer.

Fig. 4 shows an arbitrary shaped coil with 6 layers and 4 turns of each layer. Since the simulation of that coil by using the original model is time consuming, the geometrical parameter settings are adapted to the acceptable computational effort (Fig. 4). The PEC model with the obtained optimal ERP equal to 3.3531 is used. Then, the percentage error α of the presented model, the mesh time τ and the computing time t of two models related to the turns n of each layer are evaluated shown in Table I.

TABLE I
Simulation Results

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
α (%)	-3.06	1.12	-1.43	-1.39
τ of the original model (s)	1642.94	2356.77	3490.98	4232.22
τ of the presented model (s)	63.41	99.60	127.80	145.67
t of the original model (s)	119.21	186.63	231.24	312.11
t of the presented model (s)	29.13	38.97	47.38	66.23

Compared to the results of the original model, the presented model with the calculated ERP has a good accuracy and a high efficiency.

V. References

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